Beam break-up with tune chirp for an arbitrary wakefield

1997 J. Phys. A: Math. Gen. 30 8751

(http://iopscience.iop.org/0305-4470/30/24/033)

J. Phys. A: Math. Gen. **30** (1997) 8751–8760. Printed in the UK PII: S0305-4470(97)85288-2

Beam break-up with tune chirp for an arbitrary wakefield

David H Whittum
Stanford Linear Accelerator Center, Stanford University, Stanford CA 94309, USA

Received 23 June 1997

Abstract. The asymptotic form of the beam break-up instability is computed up to quadrature for an arbitrary wakefield, in the presence of a linear variation in betatron tune from head to tail along the beam. For illustration, the result is applied to a broadband impedance (resistive wall) and a narrow band impedance (single resonator) and benchmarked against simulation, for parameters of interest in induction linacs.

1. Introduction

Transverse beam break-up (BBU) has become a very popular subject over the last thirty or so years, particularly for high-current relativistic electron beams [1]. Cumulative collective instability arises for beams in plasmas [2], bunched beams in rf linacs [3], pulsed beams in induction linacs [4], in large-scale free-electron lasers [5], and in other venues [6, 7]. Many methods have been proposed to control BBU [8], and all involve one of the following: taming the wake within each cell [9, 10] or over many cells [11, 12]; taming the beam itself by tampering with the focusing mechanism thereby introducing nonlinearities [13], or a variation in focusing strength along the beam. The latter effect, a 'chirp' in betatron tune, can be accomplished via 'conditioning' [14], or by way of rf quadrupole or energy variation along the beam, a technique proposed by Balakin, Novokhatsky and Smirnov (BNS) [15]. The BNS effect in rf linacs has since been analysed by a number of workers [16, 17], and has proven invaluable for practical operation in a large collider [18].

Here the effect of tune chirp is analysed for an unbunched, coasting beam, for an arbitrary wake. The calculation is motivated by the need for a simple appraisal of tune chirp for comparison with other mechanisms, as they might apply to a large induction linac and its dominant wakefields, a resonant mode [19], and the resistive wall wake [20]. While work presented here is of special interest for intense beams, as in the 'two-beam accelerator' [21], this work is also of general interest in that it provides, for the first time, an analytic closed-form expression quantifying growth in the presence of an *arbitrary* wakefield. The problem is formulated in terms of a Green's function expressed in closed form for an arbitrary wake in section 2, and illustrated for the example of the resistive wall in section 3 and for a resonator mode in section 4.

2. Formulation

BBU is described by an equation for the transverse displacement of the beam centroid, ξ, in its simplest form [3]

$$\left(\frac{\partial}{\partial z}\gamma\frac{\partial}{\partial z} + \gamma k_\beta^2\right)\xi(z,\zeta) = \int_0^\zeta \mathrm{d}\zeta'\, W(\zeta-\zeta')\nu(\zeta')\xi(z,\zeta') \tag{1}$$

where $\zeta = t - z/c$ is the displacement along the ultrarelativistic beam and varies from 0 at the beam head to τ at the beam tail, with τ the pulse length, t the time, z the axial displacement and c the speed of light. Beam electrons remain at a fixed ζ, as they propagate in z down the beam pipe. The wake $W(\zeta - \zeta')$, is the Green's function which determines the Lorentz force on an electron at ζ due to the electric and magnetic fields generated by the beam segments at ζ'. The Budker parameter $\nu = I/I_0$ where the beam current is I and is assumed constant for $\zeta > 0$ ('unbunched beam') and zero for $\zeta < 0$. The Alfven constant $I_0 \sim 17$ kA. The Lorentz factor γ is assumed constant in z ('coasting beam'). The betatron wavenumber is assumed to take the form $k_\beta(\zeta) = k_0 + \Delta k(\zeta/\tau)$, where $k_0 = 2\pi/\lambda_0$, and λ_0 is the betatron wavelength at the beam head.

With k_β varying along the beam, the solution of equation (1) appears intractable in general. With a linear chirp however, the problem can be solved in closed form. We express ξ in terms of a complex envelope χ,

$$\xi(z, \zeta) = \mathrm{Im}\{\chi(z, \zeta) \exp(\mathrm{i}k_\beta(\zeta)z)\} \tag{2}$$

and consider the limit where the envelope χ varies little on the λ_0 length scale, i.e. the 'strong focusing' limit, $L_\mathrm{g} \gg \lambda_0$, with L_g the instability growth length. In this case an eikonal approximation is appropriate; substituting equation (2) for ξ in equation (1), we obtain

$$\frac{\partial\chi}{\partial z}(z, \zeta) \approx \frac{\nu}{2\mathrm{i}\gamma_0 k_0} \int_0^\zeta \mathrm{d}\zeta'\, W(\zeta - \zeta')\chi(z, \zeta') \exp\left(-\mathrm{i}\Delta k z \frac{\zeta - \zeta'}{\tau}\right) \tag{3}$$

and Laplace transforming equation (3) in ζ, we find

$$\frac{\partial\tilde{\chi}}{\partial z}(z, p) = \frac{\nu}{2\mathrm{i}\gamma_0 k_0} \tilde{W}\left(p + \mathrm{i}\frac{\Delta k z}{\tau}\right) \tilde{\chi}(z, p) \tag{4}$$

where p is the Laplace transform variable and the tilde denotes the Laplace transform. When $\Delta k = 0$, frequency components on the beam are continuously driven by their counterpart in the impedance. When $\Delta k \neq 0$, and after travelling a distance z, the beam will have sampled the impedance over a bandwidth $\Delta k z/\tau$.

Integrating equation (4) in z and inverting the Laplace transform produces the solution

$$\chi(z, \zeta) = \int_0^\zeta \mathrm{d}\zeta'\, G(z, \zeta - \zeta'; \Delta k)\chi(0, \zeta') \tag{5}$$

where the Green's function is given by

$$G(z, \zeta; \Delta k) = \frac{1}{2\pi\mathrm{i}} \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \mathrm{d}p\, \exp\theta(z, \zeta, p; \Delta k) \tag{6}$$

the contour is to the right of all poles of the integrand in the complex p-plane, and the exponential phase is

$$\theta = p\zeta + \frac{\nu}{2\mathrm{i}\gamma_0 k_0} \int_0^z \mathrm{d}z'\, \tilde{W}\left(p + \mathrm{i}\frac{\Delta k z'}{\tau}\right). \tag{7}$$

The asymptotic dependence of χ on z and ζ is determined by the method of steepest descent from the point(s) of stationary phase in the p-plane, where

$$\theta^{(1)}(p) = \zeta - \frac{\nu\tau}{2\gamma_0 k_0 \Delta k} \left\{\tilde{W}\left(p + \mathrm{i}\frac{\Delta k z}{\tau}\right) - \tilde{W}(p)\right\} = 0 \tag{8}$$

and $({}^{(n)} = \partial^n/\partial p^n)$. The asymptotic form for χ may then be obtained from equation (5) and depends on the character of the dominant stationary phase point(s). The contribution from a single such point is

$$\chi(z, \zeta) \approx \tilde{\chi}(0, p)(2\pi\theta^{(2)})^{-1/2} \exp\theta \tag{9}$$

when this is not an inflection point, and where θ and $\theta^{(2)}$ are evaluated at the stationary point. In the case of an inflection point one has

$$\chi(z,\zeta) \approx \tilde{\chi}(0,p)\frac{\Gamma(\frac{1}{3})}{2^{2/3}3^{1/6}\pi}(\theta^{(3)})^{-1/3}\exp\theta \tag{10}$$

where $\Gamma(\frac{1}{3}) \sim 2.6789$. In general the amplitude may include a sum over stationary points.

It will be helpful to note that if $p(z,\zeta : \Delta k)$ is a stationary point satisfying equation (8), then

$$p(z,\zeta;-\Delta k) = p(z,\zeta;\Delta k) + \mathrm{i}\frac{\Delta kz}{\tau} \tag{11}$$

is a stationary point of equation (8) with the opposite sign of Δk. Substituting this in equation (7) (with the sign of Δk reversed) one can show that the real part of the asymptotic growth exponent is *independent* of the sign of Δk. Moreover one can show that $|\theta^{(n)}|$ is also independent of the sign of Δk. Consequently asymptotic growth, as given by expressions such as equation (9) depends on the sign of Δk only through the algebraic dependence on the incident beam spectrum. To illustrate these considerations and to derive some practical scalings, we analyse two practical examples.

3. Resistive wall

We consider the impedance due to a resistive pipe [22],

$$W(\zeta-\zeta') \approx \frac{4}{\sqrt{\pi}}\frac{1}{\tau_{\mathrm{D}}^{1/2}b^2}\frac{1}{\sqrt{\zeta-\zeta'}} \tag{12}$$

where $\tau_{\mathrm{D}} = 4\pi\sigma b^2/c^2$, is a diffusion timescale, the pipe conductivity is σ, and b is the pipe radius†. It is convenient to introduce a dimensionless wakefield parameter

$$w = \frac{4}{\pi^{1/2}}\left(\frac{\nu}{\gamma}\right)\frac{1}{k_0^2b^2}\left(\frac{\tau}{\tau_{\mathrm{D}}}\right)^{1/2}. \tag{13}$$

The envelope expressed as a function of k_0z and ζ/τ is parametrized by $\Delta k/k_0$ and w. For example, for $\Delta k/k_0 \to 0$ the envelope varies with exponent

$$\mathrm{Re}\,\theta \approx \frac{3\pi^{1/3}}{2^{7/3}}w^{2/3}\left(\frac{\zeta}{\tau}\right)^{1/3}(k_0z)^{2/3} \tag{14}$$

and the beam tail experiences growth as $\exp(z/L_{\mathrm{g}})^{2/3}$ where $L_{\mathrm{g}} \sim 0.2\lambda_0/w$. The strong focusing approximation then limits us to the range $w < 0.2$ or so. Parameters of practical interest would be $\lambda_0 \sim 1$ m, $I \sim 1.5$ kA, $\sigma \sim 1\times10^{17}$ s^{-1}, $b \sim 2$ cm, $\gamma \sim 20$, $\tau \sim 20$ ns, and $L \sim 50$ m, and these correspond to $w \sim 1\times10^{-3}$ [20]. We will consider larger w to test the limits of strong focusing approximation.

To simplify algebraic expressions we introduce two additional parameters,

$$\mu = \left|\frac{k_0}{\Delta k}\right|^{3/2}\left(\frac{\tau}{\zeta}\right)\frac{2\pi^{1/2}w}{(k_0z)^{1/2}} \tag{15}$$

$$\delta = \frac{\Delta kz}{2\tau} \tag{16}$$

† For small ζ, this approximate expression for the wake breaks down; the actual wake vanishes at $\zeta \to 0$. This is of formal concern in evaluating expressions like equation (6); however, the input spectra we will consider do not sample such high-frequency behaviour and the approximate form will do.

We exchange the variable p for q such that $p = |\delta| q - \mathrm{i}\delta$ and let

$$r = \frac{2}{\mu}\sqrt{q^2+1}. \tag{17}$$

In terms of these quantities, the stationary point is determined by

$$\sqrt{2}r = \sqrt{q-\mathrm{i}} - \sqrt{q+\mathrm{i}} \tag{18}$$

which implies that r is a root of the quartic,

$$r^4 + \mu r^3 + 1 = 0. \tag{19}$$

In terms of r and q, the phase and second derivative are

$$\theta = \zeta(q|\delta| - \mathrm{i}\delta + \mu r|\delta|) \tag{20}$$

$$\theta^{(2)} = 4\zeta \frac{q + \frac{1}{4}\mu r}{\mu^2 r^2 |\delta|}. \tag{21}$$

Thus the root r determines the asymptotic growth, and there is only one root of equation (19) satisfying equation (18). For $\mu \ll 1$ (large z) it is approximately

$$r \approx \exp\left(-\mathrm{i}\frac{\pi}{4}\right) - \frac{\mu}{4}. \tag{22}$$

For detailed comparisons this approximate form is not adequate and we note the exact solution for $\mu < 1.75$,

$$r = \frac{1}{4}(u^{3/2} - \mu) - \frac{\mathrm{i}}{2}\left(u - \frac{1}{2}\mu^2 + \frac{\mu^3}{2u^{3/2}}\right)^{1/2} \tag{23}$$

where u is the root of the associated cubic ($u^3 - 4u - \mu^2 = 0$) satisfying $u > 2$, i.e. $u = 4\sin(\varphi - \pi/6)/3^{1/2}$ with $\cos(3\varphi) = 3^{3/2}\mu^2/16$ and $\pi/2 < \varphi < 5\pi/6$. (In practice, faced as one may be, with many quartics, it is simplest to rely on a quartic solving subroutine.)

As expected, the sign of the tune chirp does not appear in the real part of the exponent. Nevertheless growth does depend markedly on this sign. Taking a unit displacement of the beam at $z = 0$ for illustration, we have $\tilde{\chi}(0, p) = \mathrm{i}/p$, with $p = q|\delta| - \mathrm{i}\delta$, and considering the small μ limit, we find

$$|\chi| \approx \frac{1}{\sqrt{\pi\varepsilon}} \exp\left(\sqrt{\frac{z}{L}} - \varepsilon\right) \times \begin{cases} 1 & \Delta k < 0 \\ \dfrac{\mu^2}{16} & \Delta k > 0 \end{cases} \tag{24}$$

where

$$\varepsilon = \frac{\pi}{4} w^2 \left(\frac{\tau}{\zeta}\right)\left(\frac{k_0}{\Delta k}\right)^2 \tag{25}$$

$$L = \frac{1}{\pi^2 w^2}\left|\frac{\Delta k}{k_0}\right| \lambda_0. \tag{26}$$

To check these results, we solve equation (1) numerically. In figure 1 are depicted analytic and numerical solutions for χ at the beam tail as a function of z, for wake-strength parameter $w = 0.1$ and fractional tune chirps $\Delta k/k_0 = \pm 0.05, 0$. Clearly the amplitude is little reduced for negative tune chirp. In figure 2, comparison is made of analytic and numerical results, for several wake strengths w, as a function of $\Delta k/k_0$. (Comparison is made at the beam tail, since for these parameters, the maximum in amplitude along the beam is quite close to that at the tail.) The gradual divergence for large w of the analytic result

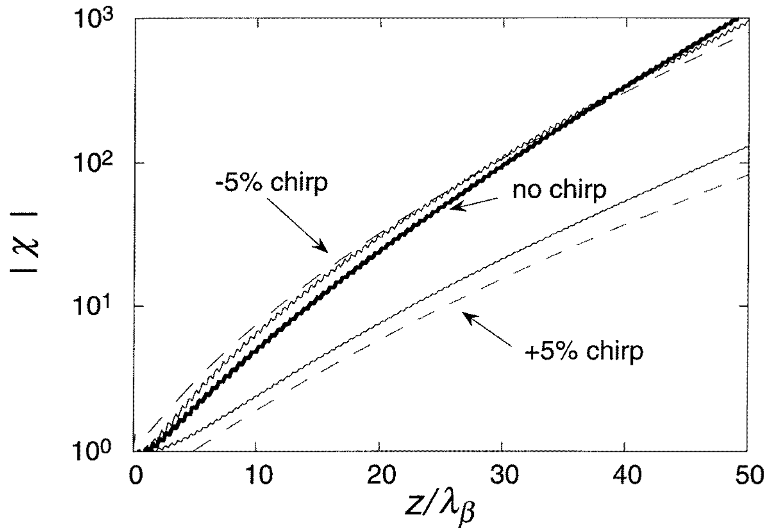


Figure 1. Comparison of the numerical solution of equation (1) and the analytic result of equation (9) for $|\chi(z, \tau)|$ as a function of z, for wake-strength parameter $w = 0.1$ and fractional tune chirps $\Delta k/k_0 = \pm 0.05, 0$.

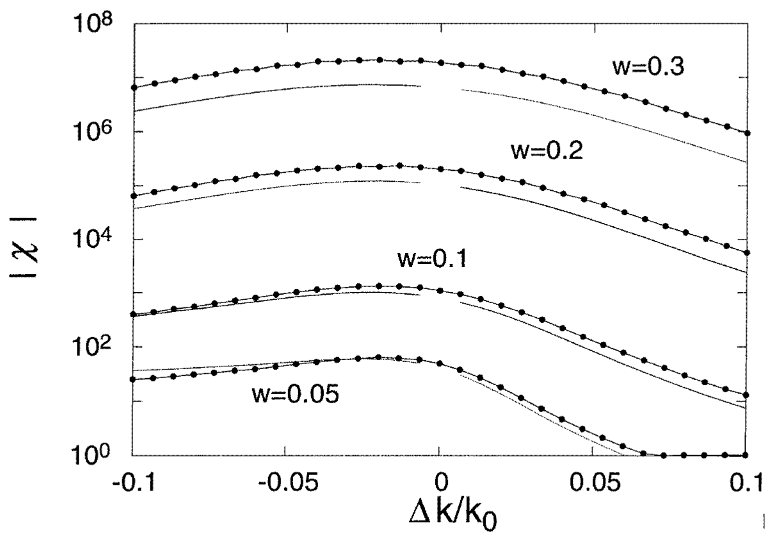


Figure 2. Comparison of the numerical solution for the maximum in $|\chi(z, \tau)|$ over the course of $50\lambda_0$, with the analytic result for $z = 50\lambda_0$, for several wake strengths w, as a function of fractional tune chirp along the beam $\Delta k/k_0$.

is due to the breakdown of the strong focusing approximation, and confirms the constraint $w < 0.2$.

A similar analysis shows that, in general, for impedances varying asymptotically as p^{-r}, with $r < 1$, there occurs a transition, from an exponent varying as z^n, with $n = 1/(1 + r)$, to one varying as $z^{(1-r)}$, representing more gradual growth. There is *no saturation* in the case of such idealized 'broadband' impedances.

4. Resonator wake

At the opposite extreme from the broadband resistive wall impedance is the resonator wake of the generic form

$$W(\zeta) = W_0 \frac{\omega_0^2}{\Omega} \exp\left(-\frac{\omega_0 \zeta}{2Q}\right) \sin(\Omega\zeta) \tag{27}$$

for which the Laplace transform is

$$\tilde{W}(p) = W_0 \frac{\omega_0^2}{\omega_0^2 + p\frac{\omega_0}{Q} + p^2} \tag{28}$$

with $\Omega = \omega_0(1 - (1/4Q^2))^{1/2}$, and Q the quality factor. Such a wake appears in the case of a single, dominant, TM_{11} mode of a microwave cavity for which the factor W_0 may be expressed in terms of a shunt impedance per unit length $r_\perp$ [23], $W_0 = r_\perp \omega_0 / Q$. In the absence of tune chirp, the solution as a function of $k_0 z$ and $\omega_0 \zeta$ is characterized by two parameters, the quality factor Q and the dimensionless wake amplitude,

$$w = \frac{\nu W_0 \omega_0}{\gamma_0 k_0^2 \Omega}. \tag{29}$$

Parameters of practical interest would be $\lambda_0 \sim 1$ m, $I \sim 1.5$ kA, $\gamma \sim 20$, $\tau \sim 20$ ns, $Q \sim 6$, $\omega_0/2\pi \sim 1$ GHz, $r_\perp/Q \sim 8\ \Omega\ \mathrm{m}^{-1}$ and $L \sim 50$ m, and these correspond to $w \sim 4 \times 10^{-3}$. As noted in previous works there are several regimes of growth [1] and we will specialize to the limit of a long pulse $\Omega\tau \gg 1$. In this case, with no tune chirp, the asymptotic exponent is

$$\mathrm{Re}\,\theta = (k_0 z \omega_0 \tau w)^{1/2} - \frac{\omega_0 \tau}{2Q} \tag{30}$$

corresponding to a growth length $L_\mathrm{g} \sim \lambda_0 / 2\pi w \omega_0 \tau$. The condition for adiabatic growth is $w < 1/(2\pi\omega_0\tau)$, or $w < 1 \times 10^{-3}$ for the 'typical' parameters. We will consider $w \sim 1 \times 10^{-2}$ to test the limits of the strong focusing approximation.

To account for the chirp it is convenient to introduce the dimensionless parameters

$$\delta = \frac{\Delta k z}{2\Omega\tau} \tag{31}$$

$$\varepsilon = \frac{\nu W_0 \omega_0^2}{4\gamma_0 \Omega} \frac{1}{\Delta k} = \frac{1}{4 \Delta k L_\mathrm{g}}. \tag{32}$$

The term ε is small when phase mixing is rapid on the scale of a growth length. We make the change of variable,

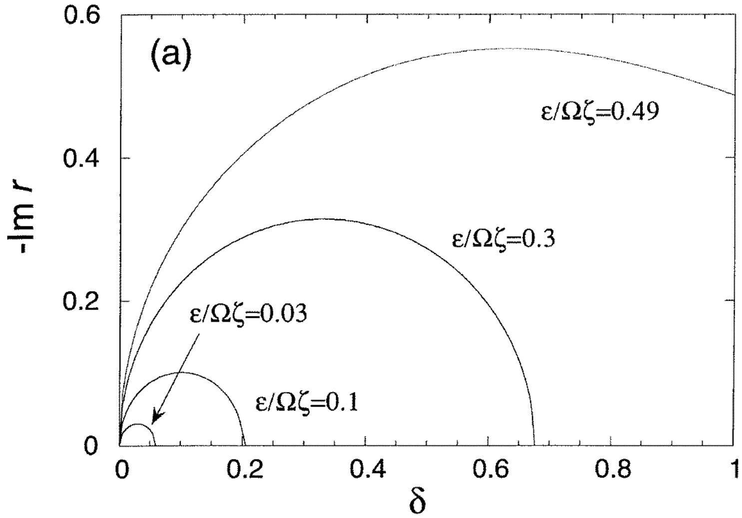
$$p = \mathrm{i}\Omega(r - \delta) - \frac{\omega_0}{2Q} \tag{33}$$

in terms of which the exponent in equation (7) takes the form

$$\theta = -\mathrm{i}\delta\Omega\zeta - \frac{\omega_0}{2Q}\zeta + \mathrm{i}\Omega r + \mathrm{i}\varepsilon \ln\left(\frac{1 - r - \delta}{1 - r + \delta} \frac{1 + r - \delta}{1 + r + \delta}\right) \tag{34}$$

with the logarithm defined by analytic continuation from the region of r imaginary and negative. After differentiation and some algebra, equation (8) results in a two-parameter quartic polynomial in r,

$$r^4 - 2r^2(1 + \delta^2) - 8r\delta\left(\frac{\varepsilon}{\Omega\zeta}\right) + (1 - \delta^2)^2 = 0 \tag{35}$$



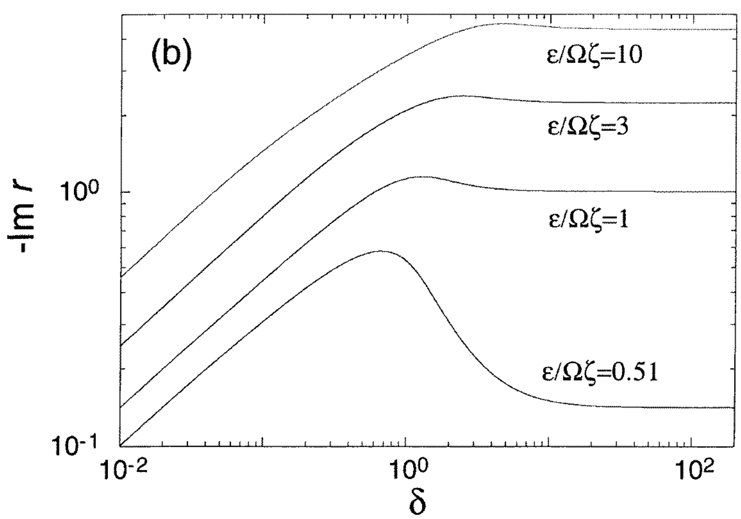


Figure 3. Solutions of equation (35) for the root r determining asymptotic growth in the case of a resonator wake, as a function of δ. Solutions correspond to (a) $\varepsilon/\Omega\zeta < \frac{1}{2}$ and (b) $\varepsilon/\Omega\zeta > \frac{1}{2}$. For small $\varepsilon/\Omega\zeta$ saturation ($\mathrm{Im}\, r \to 0$) occurs for $\delta \sim 2\varepsilon/\Omega\zeta$. For large $\varepsilon/\Omega\zeta$, $\mathrm{Im}\, r \to -(2\varepsilon/\Omega\zeta)^{1/2}$, for $\delta \sim 2$–3.

with coefficients independent of the sign of the tune chirp. Only one root of this quartic results in growth; illustrative solutions for $\mathrm{Im}\, r$ are depicted in figure 3.

It is instructive to consider explicit analytical scalings. For small ζ such that $\varepsilon/\Omega\zeta \gg 1$, $\mathrm{Re}\, r \sim -|\delta|$, $\mathrm{Im}\, r \sim -(2|\varepsilon|/\Omega\zeta)^{1/2}$ and the influence of the mode resonance at Ω is diminished. Effectively the wake is linear and the impedance varies as $1/p^2$. In this case saturation occurs after a range $|\delta| \sim 2$–3, usually well past the range of interest $\sim 50\lambda_0$. Accurate prediction of the amplitude in the intermediate regime, prior to saturation, requires the exact solution of the quartic.

More typically one would be interested in the limit $\varepsilon/\Omega\zeta$ small; this corresponds, at the beam tail, to the condition $|\Delta k|/k_0 > w$, a chirp of perhaps 1% or more. In this

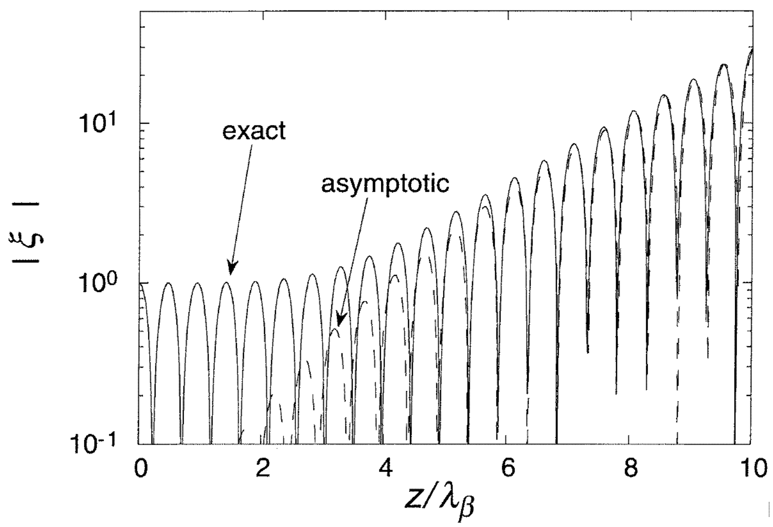


Figure 4. Depicted is the result of equation (9) (broken curve) overlayed with the simulation for a +5% chirp, for a pulse 20 resonator periods in length, $Q = 6$ and wake strength $w = 0.01$. Evidently the asymptotic form converges after just a few betatron periods.

limit $\mathrm{Im}\, r$ vanishes beyond $\delta \sim 2\varepsilon/\Omega\zeta$, corresponding to a inflection point in the phase lying just short of the point of maximum amplitude. This provides a rough estimate of the length for saturation of the beam tail $z_{\mathrm{sat}} \sim 1/\Delta k^2 L_{\mathrm{g}}$. As one can see from equation (33) (taking δ small, and $r \sim -1$) the root p does not exhibit a strong asymmetry in this limit, since regardless of the sign of the chirp $-\mathrm{Im}\, p$ lies near Ω. To obtain an estimate of this amplitude at saturation we apply equation (10) and after some algebra find

$$|\chi|_{\mathrm{sat}} \approx 0.6 \frac{|\varepsilon^{2/3}|}{\Omega\zeta} \exp\left(\pi|\varepsilon| - \frac{\omega_0 \zeta}{2Q}\right). \tag{36}$$

To check these results we may again solve equation (1) numerically. Illustrative results are shown in figures 4–6. Parameters are fixed at $\tau = 40\pi/\omega_0$, and $Q = 6$. Figure 4 compares the asymptotic result of equation (9) with simulation for a +5% chirp and $w = 1 \times 10^{-2}$, indicating that close agreement can be obtained after just a few betatron periods.

Figure 5 compares the asymptotic form with rms envelopes from the simulation, versus position for ±5% and 0 chirp, with $w = 1 \times 10^{-2}$. One can see that there is only a small asymmetry in chirp; one can also see the inflection point just preceding saturation (the small peaks, where equation (9) breaks down). In figure 6 the maximum in amplitude at the beam tail in the course of propagation through $50\lambda_0$ is depicted versus chirp, for several different wake amplitudes w. Overlayed are the corresponding analytic results: equation (10) when $z_{\mathrm{sat}} < 50\lambda_0$, otherwise equation (9) evaluated at $50\lambda_0$. The broken curves are the maximum in amplitude over the entire beam, and show that for larger chirps the peak amplitude occurs within the body of the pulse. Evidently there is no dramatic dependence on the sign of the chirp.

5. Conclusions

BBU growth has been computed up to quadrature for an arbitrary wake in the presence of a linear tune chirp. This result reduces the problem of computing asymptotic growth to that of

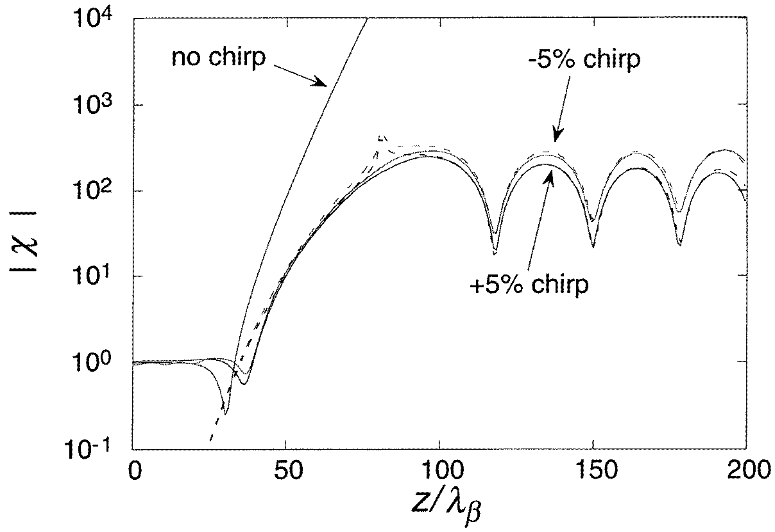


Figure 5. Shown are the asymptotic forms for the envelopes from the simulation, versus position for ±5% and 0 chirp, with $w = 0.01$ and $Q = 6$. Overlayed are the results of equation (9) (broken curves). The small peaks on the analytic curves (just preceding saturation) are in the vicinity of the inflection point where equation (9) breaks down.

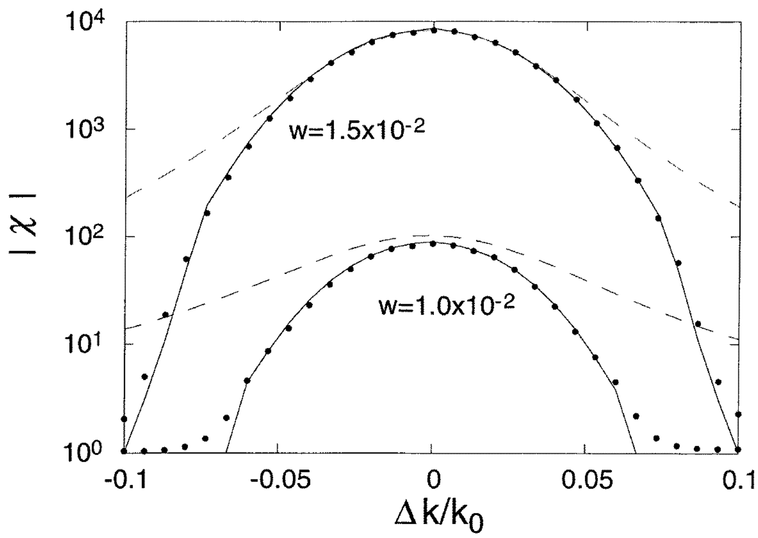


Figure 6. Shown here is a survey of the maximum in amplitude over the course of propagation through $50\lambda_0$ versus chirp, for several different wake amplitudes w, with $Q = 6$. Overlayed is the analytic prediction, either the saturated or unsaturated result, as explained in the text. The slopes on the curves change slightly at the transition from saturation within $50\lambda_0$ to unsaturated.

identifying and characterizing stationary points. The result was applied to two representative practical examples, the broadband resistive wall impedance, and the narrow-band resonator impedance.

In the case of the broadband impedance we saw that tune chirp does not in general produce saturation as one would expect from a damping mechanism (namely Landau damping). For a broadband impedance, varying as $1/p^r$ with $r < 1$ no saturation results, although growth can be drastically diminished. In the case of a resonator mode, saturation always results, although the form of the amplitude differs depending on the size of the chirp $|\Delta k|/k_0$ versus the wake amplitude w.

Acknowledgment

This work has benefitted from helpful conversations with Professor Alex Chao.

References

[1] Lau Y Y 1989 *Phys. Rev. Lett.* **63** 1141
[2] Lee E P 1978 *Phys. Fluids* **21** 1327
Lampe M, Joyce G, Slinker S P and Whittum D H 1993 *Phys. Fluids* B **5** 1888
[3] Chao A W 1993 *Physics of Collective Beam Instabilities in High Energy Accelerators* (New York: Wiley)
[4] Neil V K, Cooper N and Cooper R K 1970 *Part. Accel.* **1** 111
Caporaso G J, Barletta W A and Neil V K 1980 *Part. Accel.* **11** 71
[5] Rangarajan G and Chan K C D 1989 *Phys. Rev.* A **39** 4749
[6] Nguyen K T, Schneider R F, Smith J R and Uhm H S 1987 *Appl. Phys. Lett.* **50** 239
[7] Bosch R A, Menge P R and Gilgenbach R M 1992 *J. Appl. Phys.* **71** 3091
[8] Wilson P B 1989 Physics of Particle Accelerators *AIP Conf. Proc.* vol 184, ed M Month and M Dienes (New York: AIP) pp 525–64
[9] Yu D and Kroll N 1991 *Conf. Record of the 1991 IEEE Particle Accelerator Conference* (New York: IEEE) pp 1716–18
[10] Menge P R, Gilgenbach R M and Lau Y Y 1992 *Phys. Rev. Lett.* **69** 2372
[11] Yu D and Kim J S 1991 *Conf. Record of the 1991 IEEE Particle Accelerator Conference* (New York: IEEE) pp 1719–21
[12] Takayama K 1992 *Phys. Rev.* A **45** 1157
[13] Caporaso G J, Rainer F, Martin W E, Prono D S and Cole A G 1986 *Phys. Rev. Lett.* **57** 13
[14] Fernsler R F, Hubbard R F and Slinker S P 1992 *Phys. Fluids* B **4** 4153
[15] Balakin V E, Novokhatsky A V and Smirnov V P 1984 *Proc. 12th Int. Conf. on High-energy Accelerators* ed F T Cole and R Donaldson (Batavia: Fermi National Accelerator Laboratory) pp 119–20
[16] Thompson K A and Ruth R D 1990 *Phys. Rev.* D **41** 964
[17] Spence W L 1993 BNS Damping-'Autophasing' and Discrete Focusing *Proc. Int. Workshop on Emittance Preservation in Linear Colliders (KEK Con. Proc. 93-13)* ed J Urakawa and K Oide (Tsukuba: KEK) pp 510–13
[18] Seeman J T, Decker F-J, Holtzapple R L and Spence W L 1994 *Proc. 1993 IEEE Particle Accelerator Conf.* (New York: IEEE) pp 3234–6
[19] Kim J S, Henke H, Sessler A M and Whittum D H 1994 *Proc. 1993 IEEE Particle Accelerator Conf.* (New York: IEEE) pp 3288–90
[20] Whittum D H, Sessler A M and Neil V K 1991 *Phys. Rev.* A **43** 294
[21] Li H, Houck T, Goffeney N, Henestroza E, Sessler A, Westenskov G and Yu S 1995 Design study of beam dynamics issues for a one TeV next linear collider based upon the relativistic Klystron two-beam accelerator *Advanced Accelerator Concepts (AIP Conf. Proc. 335)* ed P Schoessow (New York: AIP) pp 817–36
[22] Bodner S, Neil V K and Smith L 1970 *Part. Accel.* **1** 327
[23] Briggs R J, Birx D L, Caporaso E J, Neil V K and Genomi T C 1985 *Part. Accel.* **18** 41